RESEARCH AND DEVELOPMENT BRANCH
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DREO TECHNICAL NOTE NO. 75-19
DREO TN 75-19

HEAT AND MOISTURE TRANSFER IN CLOTHING SYSTEMS. PART 2. THEORETICAL CONSIDERATION OF THE EFFECT OF SOME VARIABLES ON THERMAL CONDUCTIVITY

by
R.M. Crow

PROJECT NO.
79-01-04

OCTOBER 1975
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9 Technical note,

14 DREO-TN-75-19

11 Oct 75

12 24p.

PROJECT NO.
79-01-04

RECEIVED OCTOBER 1975
PUBLISHED OCTOBER 1975
OTTAWA

75-194

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ABSTRACT

The theoretical effect of several variables on the total thermal conductivity of a fibrous system was determined using derived mathematical relationships. The variables examined included percentage fibre content, fibre arrangement, environmental temperature, water, ice and water vapour diffusion. It was found that the presence of water or ice greatly increases thermal conductivity, and that at practical fibre contents, minimal thermal conductivity is attained when the fibres are lying in series to the direction of heat flow.

RÉSUMÉ

L'effet théorique de plusieurs variables sur la conductivité thermique totale d'un système fibreux a été déterminé à l'aide de rapports mathématiques dérivés. Les variables étudiées comprenaient la teneur en fibres exprimée en pourcentage, la disposition des fibres, la température ambiante et le degré de diffusion de l'eau, de la glace et de la vapeau d'eau. On a découvert que la présence d'eau ou de glace augmente de beaucoup la conductivité thermique et qu'on atteint la conductivite thermique minimale, dans les teneurs en fibres pratiques, lorsque les fibres sont étendues en series dans le sens du courant de chaleur.

1. INTRODUCTION

In Part 1 of this technical note (1), the complexities of the effect of variables on thermal insulation were discussed and the relevant literature was reviewed. Further, a theoretical equation for combined heat and moisture transfer through a fibrous system was derived. In this part of the technical note, the effects of several variables on the overall thermal conductivity of fibrous system are examined, using the derived equation. The variables examined include percentage volume of fibre, fibre arrangement, relative humidity, liquid water, ice and temperature.

The system developed in detail in Part 1 (1) and under consideration here is shown in Figure 1, where a temperature gradient $T_1 - T_2$ exists across the system. Two conditions of vapour pressure across the gradient have been selected;- namely, $P_1 = P_2$ or $P_1 > P_2$. The latter condition commonly occurs during the wearing of clothing.

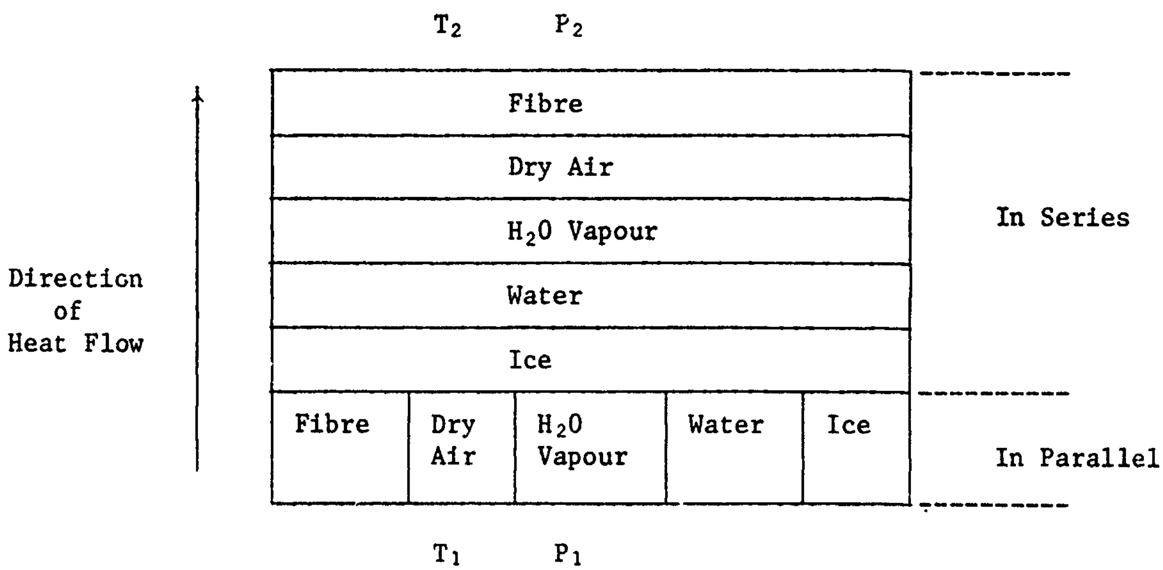

Fig. 1 Heat Flow in Total System Through Parallel and Series Components.

2. EQUATIONS USED AND LIMITS DEFINED

2.1 General Equation

The general theoretical equation for the total thermal conductivity across a fibrous system (1) referred to a textile system of the type which would be used in Arctic winter clothing. Such a system consists essentially of fibres with spaces between them; these spaces may contain moist air, water or ice. Thus, the arrangement of the fibres with respect to the direction of heat flow will determine the arrangement of each associated phase - moist air, water or ice. Hence, the general equation (A) for total conductivity includes components corresponding to each one of these phases.

Accordingly,

$$\text{(A)} \quad K_T = V_T K_T = V_s K_s + V_p K_p$$

where V_T is the total volume of the system (=1)

K_T is the effective conductivity of all phases combined

V_s is the volume fraction of all components in series to the direction of heat flow

V_p is the volume fraction of all components in parallel to the direction of heat flow.

and

$$\text{(B)} \quad V_T = V_s + V_p = 1$$

The series volume fraction is made up of volume fractions for each of the following phases; fibre (f), air (a), water (w) and ice (i). That is,

$$\text{(C)} \quad V_s = V_{fs} + V_{as} + V_{ws} + V_{is} + V_{ds}$$

The additional term V_{ds} is the volume fraction representing the effect of diffusion in the series component. Similarly for the parallel volume fraction,

$$\text{(D)} \quad V_p = V_{fp} + V_{ap} + V_{wp} + V_{ip} + V_{dp}$$

Each of these phases has a corresponding conductivity, represented by a small k (with similar subscripts). The magnitude of the conductivity for each phase is independent of the arrangement of the phase with respect to the direction of heat flow.

For the previously derived equation (1),

$$\text{(E)}\quad V_T K_T = V_s[1 \div (V_{fs}/k_f + V_{as}/k_a + V_{ws}/k_w + V_{is}/k_i + V_{ds}/k_d)] + V_p\ (V_{fp}k_f + V_{ap}k_a + V_{wp}k_w + V_{ip}k_i + V_{dp}k_d)$$

Since V_T is equal to 1, K_T can be found for arbitary values of the volume fractions and known values for the conductivities of each phase.

2.2 Fibre Phase

To reduce the complexity of the system, the fibre in the system under consideration is defined as being inert, i.e. it has constant dimensions and weight, and no water molecules penetrate into it, or adhere to its surface.

2.3 Gaseous Phase

Again, for simplification, it was assumed that conduction occurs through the gaseous phase when a temperature gradient alone exists across the system. For this state, the thermal conductivity of dry air is used (k_a) even when water vapour is present. The effect of water vapour on the thermal conductivity of an air-water vapour mixture is small since the relative amount of water vapour is very small compared to air (5% at 33°C and 100% R.H. and 0.04% at -33°C and 100% R.H.) and the thermal conductivity of water vapour differs from that of dry air by approximately 0.008 $W\ m^{-1}{}^{\circ}K^{-1}$ (33%) at any given temperature from -25 to +35°C.

When both a temperature gradient and a water vapour pressure gradient exist across the system, conduction, (k_a), is assumed through the dry air component, and diffusion with its equivalent conductivity, (k_d), is assumed through the water vapour component.

2.4 Liquid Phase

Conduction through the liquid phase (water) will occur at temperatures at or above 0°C. When the system is totally saturated with water only the fibre and liquid phases will be present.

2.5 Solids Phase

The conditions for conduction through the solid phase (ice) are similar to those through the liquid phase, except that conduction will occur at temperatures at or below 0°C.

2.6 Thermal Conductivity Equations

Since the thermal conductivities of liquid water, ice and air are temperature dependent, a least squares regression analysis for all phases except ice was used to produce lines of best fit for the relationships between temperature in °C (x) and thermal conductivity (y) of each phase in W m^{-1}°K^{-1}. An F-test showed these relationships to be linear. Since only two values of thermal conductivity of ice could be found at two temperatures (See Appendix 1), a linear relationship was assumed. The thermal conductivity of the fibre phase is taken to be six times that of dry air (2). The equation for equivalent thermal conductivity for diffusion is that reported by Nissan et al. (3). All equations are given in Appendix 1.

2.7 Conditions for Calculation of Total Thermal Conductivity

The theoretical equation (E) contains seven variables, namely percentage fibre content and arrangement of the fibres (series and parallel), environmental temperature, liquid water, ice and water vapour diffusion. The number of combinations and permutations of these variables is almost limitless. Preliminary calculations showed that some of the variables were of much greater importance than others. Accordingly, the conditions for calculation were selected as shown in Table I, from which it can be seen that fibre volume was varied for each environmental condition using both the 'in series' and 'in parallel' fibre arrangements. The methods for calculating the total thermal conductivity for any combintation of environmental or fibre conditions at the given temperature ranges are given in the results.

The temperature gradients were selected to approximate those which would occur in the wearing of clothing in cold environments. 33°C approximates body temperature, 0°C, the minimum temperature at which water remains liquid and the maximum temperature at which ice remains solid. Minus thirty three degrees Celsius puts this theoretical consideration into virgoian balance.

The water vapour gradient is taken to be the maximum possible, i.e the saturation vapour pressure at the selected mean temperature. The relative amount of the water vapour in an air-water vapour mixture is 1.85% at the mean temperature of +16.5°C, and 0.17% at -16.5°C.

TABLE I

Conditions for Calculation of Total Thermal Conductivity

Temperature Range (°C)	Mean Temperature $\bar{T}$ (°C)	V_f	V_a	V_d	V_w	V_i
+33 to 0	+16.5	0 to 1 by 0.1	$1 - V_f$	0	0	0
"	"	"	$1 - V_f - V_d$	$0.0185(1-V_f)$	0	0
"	"	"	0	0	$1-V_f$	0
-33 to 0	-16.5	"	$1 - V_f$	0	0	0
"	"	"	$1 - V_f - V_d$	$0.0017(1-V_f)$	0	0
"	"	"	0	0	0	$1-V_f$

V_sK_s and V_pK_p were calculated for each of the above conditions

3. RESULTS AND DISCUSSION

The results are given in Appendix 2, and are presented graphically in Figures 2, 4, 5, 6 and 7.

3.1 Temperature Gradient

The effect on thermal conductivity of adding air, water or ice in series and in parallel, to increasing amounts of fibre is shown in Figure 2. A linear relationship exists when the fibres are in parallel, and a non-linear relationship when the fibres are in series. The line of best fit for each curve was found by least squares regression analysis, and the equations for these curves are given in Table II.

As the percent content of fibre is increased, the total thermal conductivity decreases when water or ice is the given component in the system, and increases when it is air. These results are to be expected since the

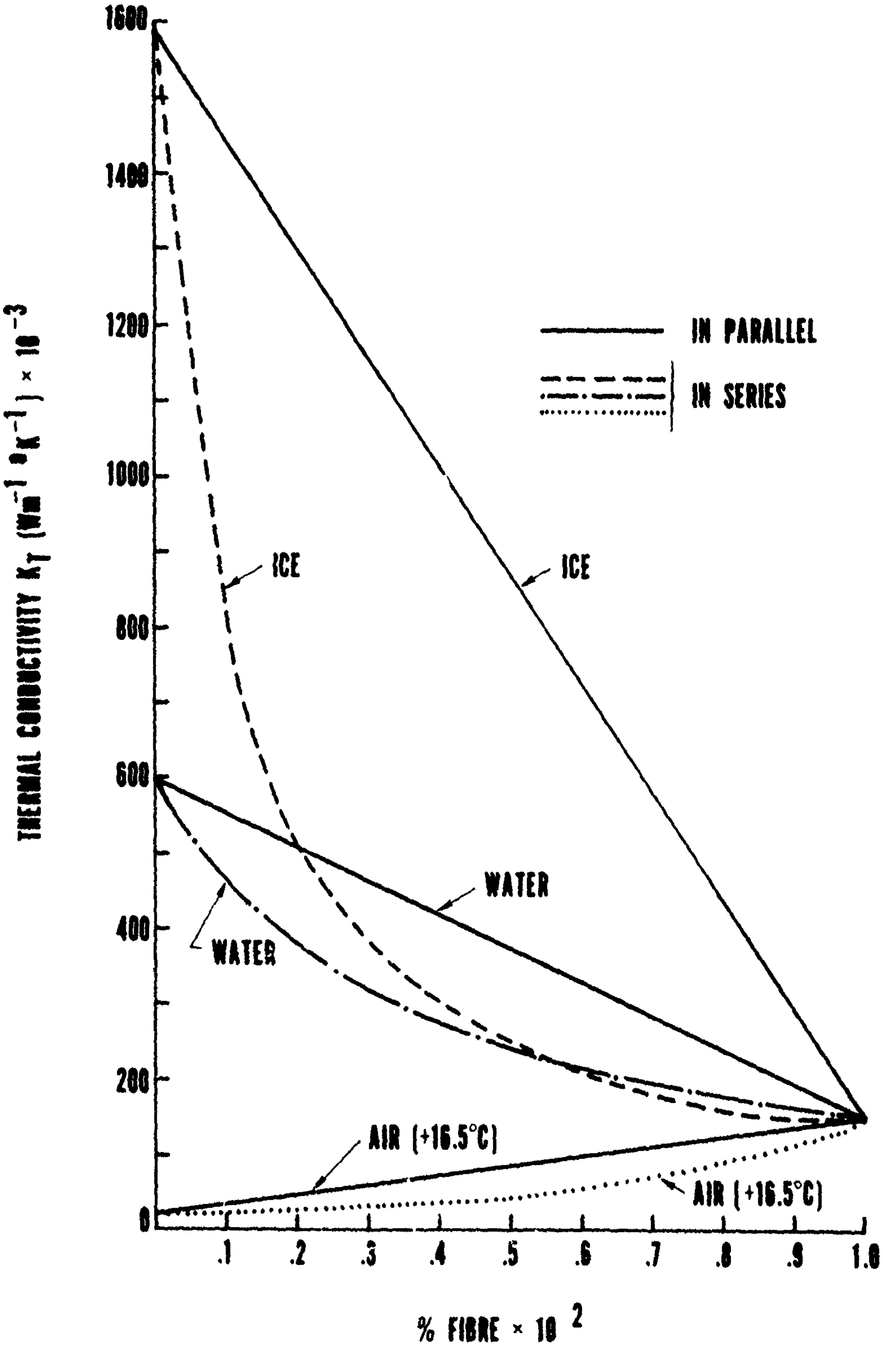

Fig. 2 The Effect of Variables on Thermal Conductivity

TABLE II

Equations of Lines of Best Fit for Temperature Gradient

Fibre and Air (+16.5°C)

Series

$$y = 0.023 + 0.098x - 0.275x^2 + 0.303x^3$$

Parallel

$$y = 0.026 + 0.127x$$

Fibre and Air (-16.5°C)

Series

$$y = 0.020 + 0.086x - 0.241x^2 + 0.266x^3$$

Parallel

$$y = 0.023 + 0.114x$$

Fibre and Water (+16.5°C)

Series

$$y = 0.589 - 1.310x + 1.559x^2 - 0.691x^3$$

Parallel

$$y = 0.597 - 0.444x$$

Fibre and Ice (-16.5°C)

Series

$$y = 0.011 + \frac{0.131}{x} - \frac{0.006}{x^2}$$

Parallel

$$y = 1.585 - 1.449x$$

thermal conductivity of the fibres is less than that of water or ice alone (by four and eleven and a half times respectively), and is greater than air (by six times).

As indicated in Figure 2, the ice-fibre in series curve crosses the water-fibre in parallel curve at 20% fibre content, and the water-fibre in series curve at approximately 55% fibre content. Since the percent volume of fibres in most textile fabrics fall within this range (i.e 20 to 55%), those environmental variables which can be expected to have the greatest effect on thermal conductivity in order of priority are shown in Figure 3.

good insulation

air/fibres in series at $\bar{T} = -16.5°C$
air/fibres in series at $\bar{T} = +16.5°C$
air/fibres in parallel at $\bar{T} = -16.5°C$
air/fibres in parallel at $\bar{T} = +16.5°C$
water/fibres in series
ice/fibres in series
water/fibres in parallel
ice/fibres in parallel

increasing thermal conductivity ↓

poor insulation

Fig. 3 Effect of Variables on the Thermal Conductivity of a Fibrous System

Figure 4 shows the effect of temperature on thermal conductivity. That is a decrease in mean temperature decreases the thermal conductivity, this decrease being greater as the percentage volume of fibre is increased. A t-test showed a significant difference at the 95% level between the thermal conductivity of an air-fibre system at +16.5 and -16.5°C for both types of fibre arrangements.

3.2 Temperature and Vapour Gradient

The inclusion of the diffusion component with the air lowers the total thermal conductivity (Figure 5 and 6) at both +16.5 and -16.5°C mean temperatures. A t-test showed these differences to be significant at the 95% level. These results disagree with the work of Nissan et al. (3) in which he concludes that heat transfer rates are increased by the evaporation-diffusion-condensation mechanism, particularly at high temperatures. The discrepancy may be due to the fact that only the diffusion part of this mechanism is considered here, and that lower temperatures were used in the present calculations.

However, if condensation of the diffusing water vapour does occur, the presence of water, or if freezing temperatures prevail, ice, will cause

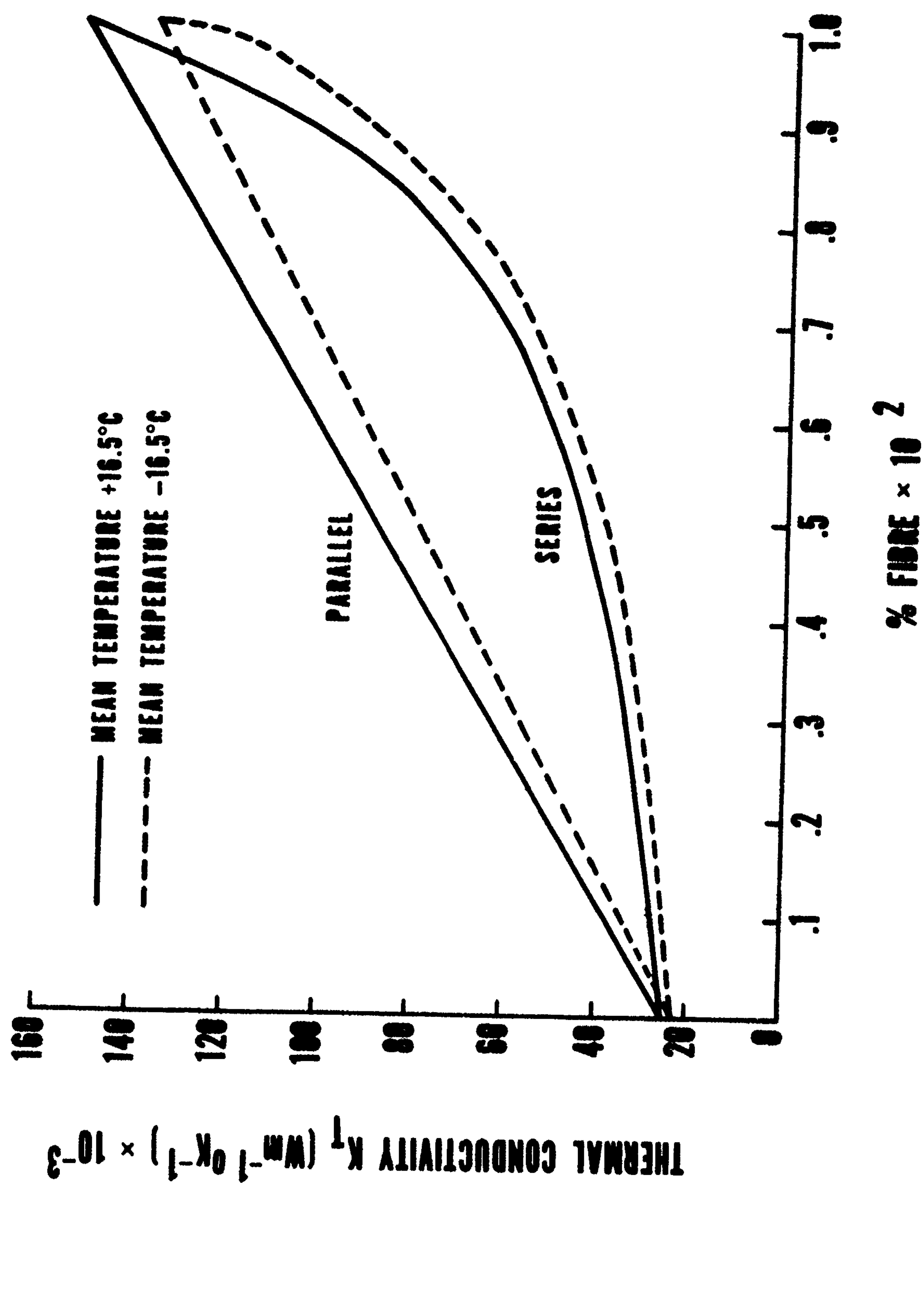

Fig. 4 Effect of Temperature on Thermal Conductivity

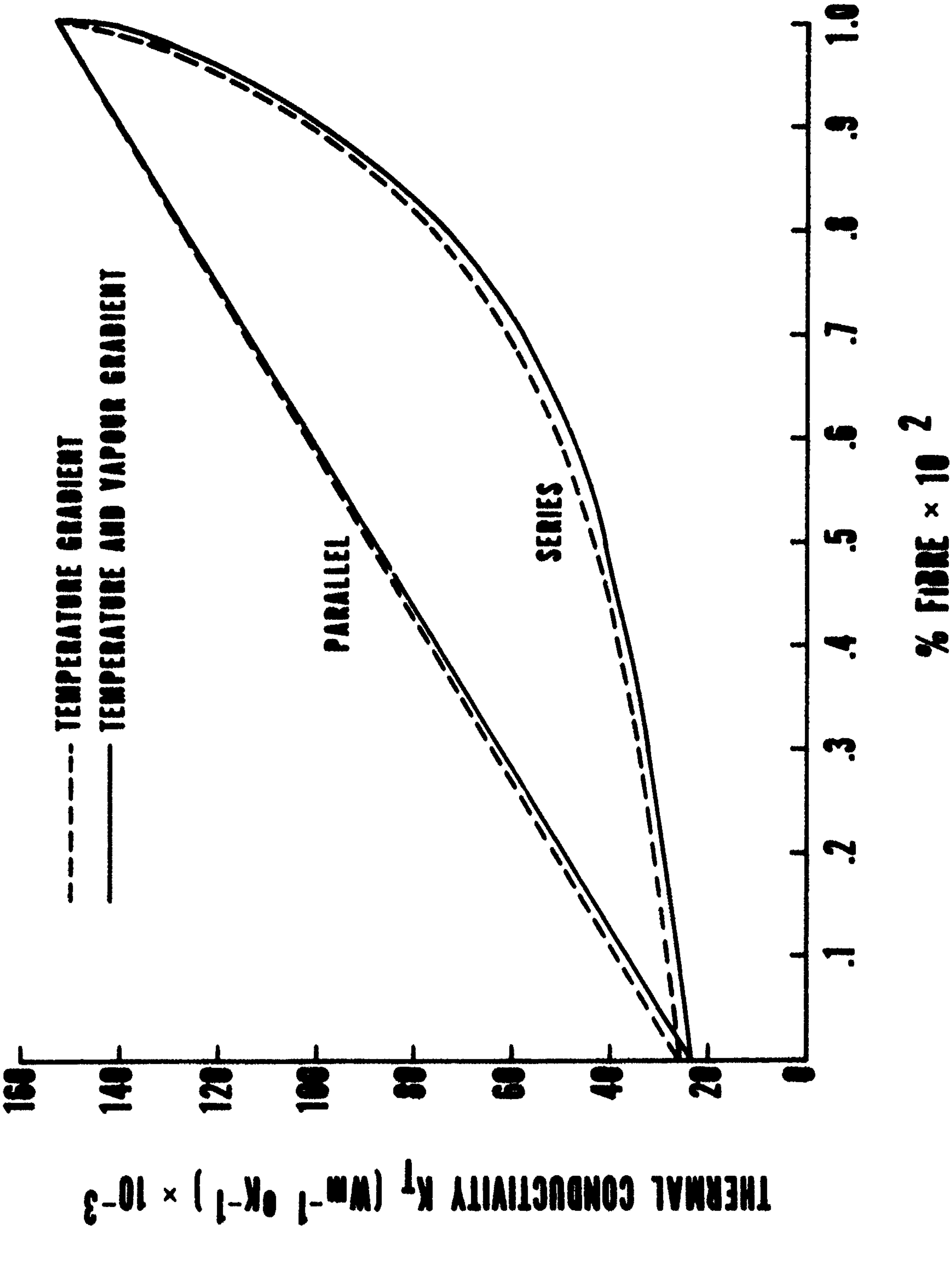

Fig. 5 The Effect of Diffusion on Thermal Conductivity at +16.5°C

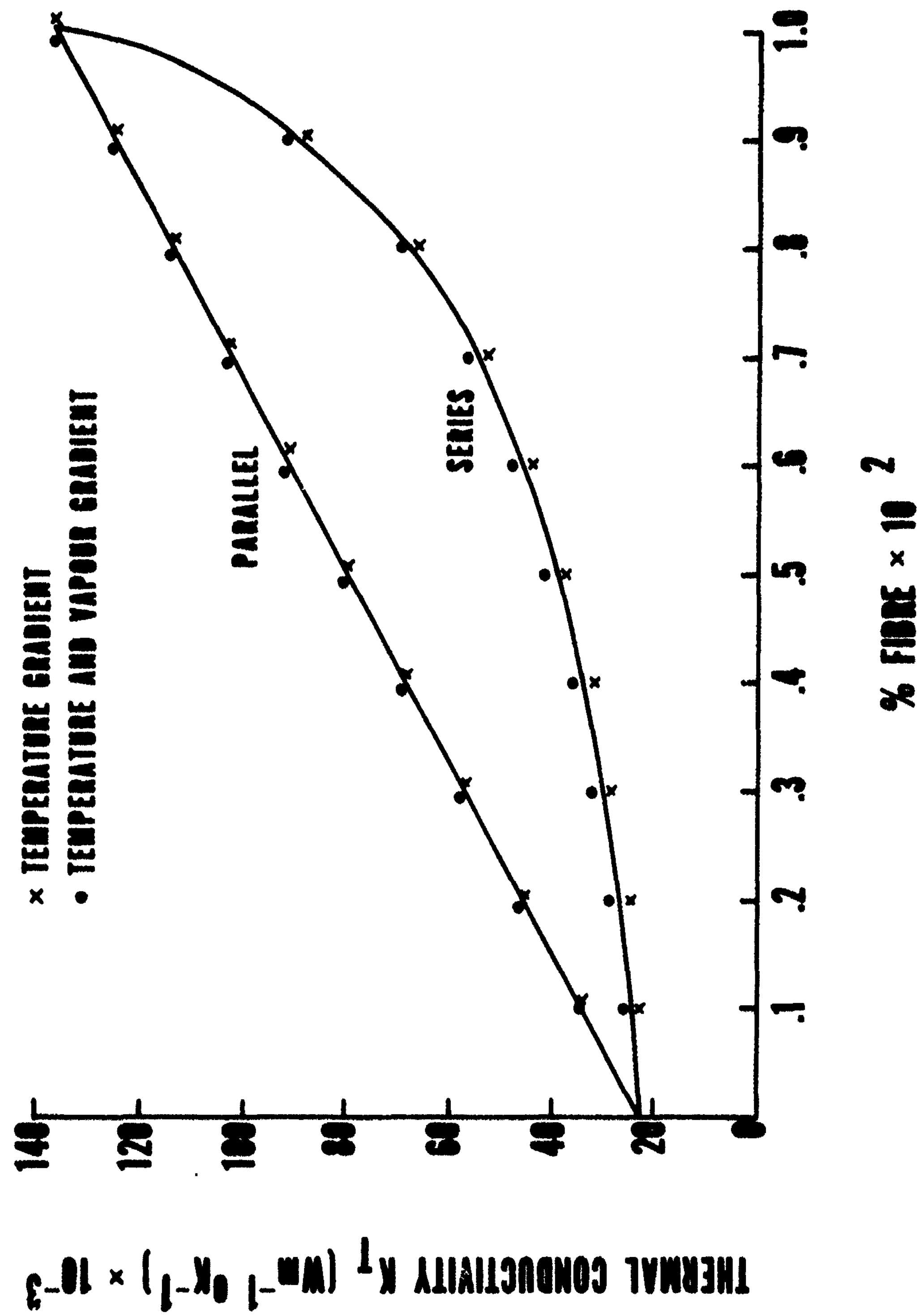

Fig. 6 The Effect of Diffusion on Thermal Conductivity at -16.5°C

a considerable increase in thermal conductivity, e.g., at 50% fibre content, the increase in thermal conductivity from air to water is approximately 4 to 6 fold, and from air to ice 6 to 12 fold for in series and in parallel respectively.

TABLE III

Equations of Lines of Best Fit for Temperature and Vapour Gradients

Fibre and Air at +16.5°C

In Series

$$y = 0.021 + 0.103x - 0.248x^2 + 0.272x^3$$

In Parallel

$$y = 0.025 + 0.128x$$

Fibre and Air at -16.5°C

In Series

$$y = 0.020 + 0.088x - 0.248x^2 + 0.272x^3$$

In Parallel

$$y = 0.023 + 0.114x$$

3.3 Thermal Conductivity for Combination of Variables

Computation from the conductivity equation of the total thermal conductivity of a system which includes any combination of the variables under consideration here would be a complex and tedious task. Information is more readily found graphically, or by solving the equations for the regression lines.

Using the data illustrated in Figure 7, for example, consider a system which is 40% fibre, 30% air and 30% water with half of the fibers lying in series, and the other half lying in parallel to the direction of the heat flow. The thermal conductivity will lie on the line x = 0.4. If there is only air present, the thermal conductivity will be that value at point A midway between the 'in series' and 'in parallel' curves for air. Similarly, if there is only water present, the thermal conductivity will lie at point B, the midpoint between the 'in series' and 'in parallel' curves for water. If the remaining 60% space is equally occupied by air and water, the total thermal conductivity for the example will be at point C midway between B and A. The thermal conductivity value at this point is 0.200 W m^{-1}°K^{-1}

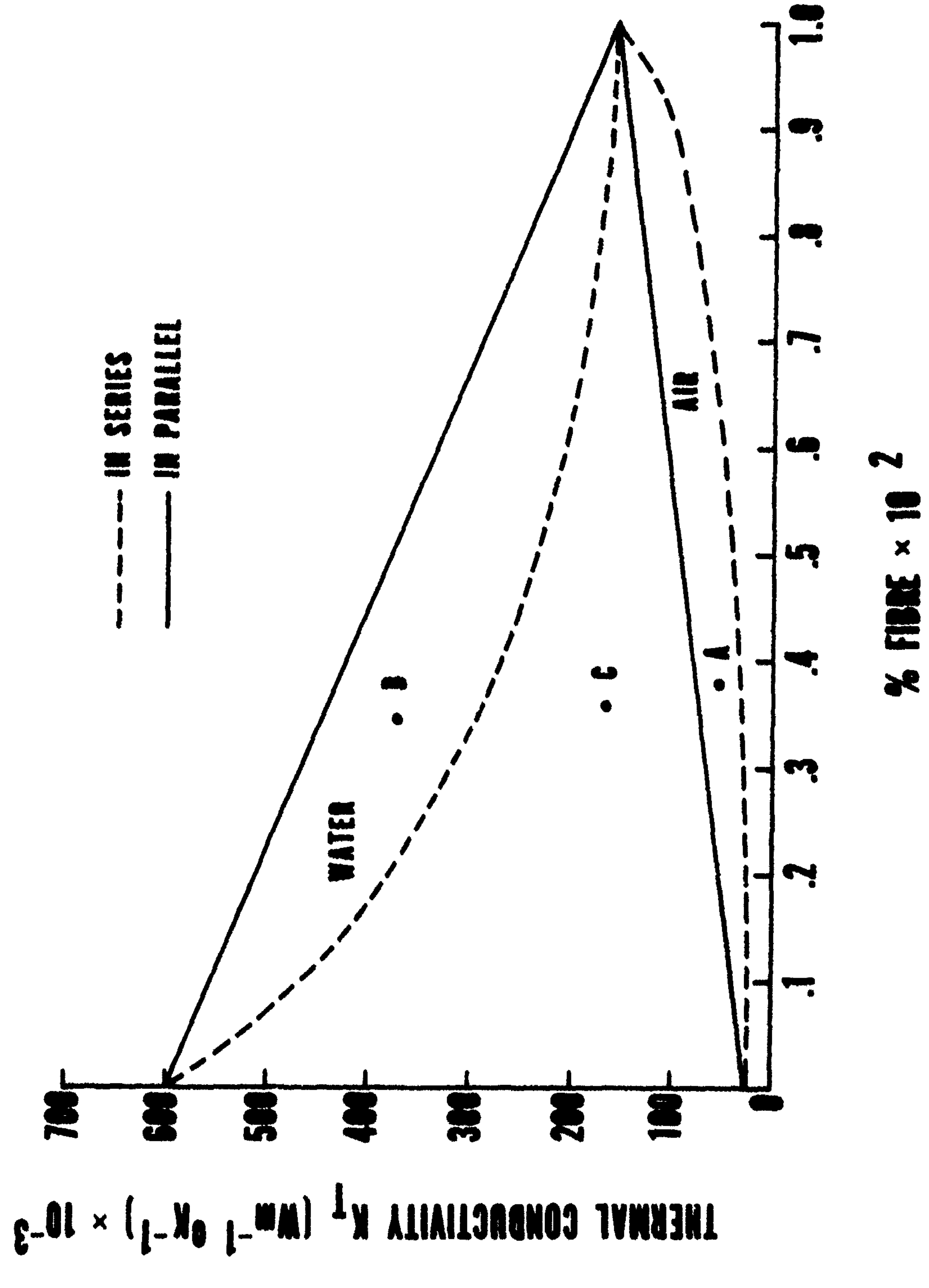

Fig. 7 Graphic Solution of Thermal Conductivity for a Combination of Variables

To solve the example using the regression equations, the values of thermal conductivity for air and water 'in series' and 'in parallel' at 40% fibre content are calculated. These are in W m^{-1}°K^{-1}

air in series	= 0.038
air in parallel	= 0.077
water in series	= 0.270
water in parallel	= 0.419

The values at Points A and B would be 0.058 and 0.344 respectively, to give the value at C as 0.201 W m^{-1}°K^{-1}.

4. CONCLUSIONS

1. Minimum thermal conductivy occurs for a fibre and air system in which the fibres are lying in series to the direction of heat flow.
2. The more still air in the system, the less the thermal conductivity.
3. The addition of water or ice to the system greatly increases the thermal conductivity of the system.
4. A clothing system having fibres arranged in series to the direction of heat flow will maintain thermal insulation better if moisture accumulation occurs, than a similar system in which the fibres are arranged parallel to the direction of heat flow.
5. Heat transfer by diffusion has very little effect of the thermal insulation in the range of temperatures considered.

5. REFERENCES

1. Heat and Moisture Transfer in Clothing Systems Part 1. Review of Literature. R.M. Crow (1974) DREO Tech. Note 74-27.

2. W.E. Morton and J.W.S. Hearle, Physical Properties of Textile Fibres, The Textile Institute (1962).

3. Heat Transfer in Porous Media Containing a Volatile Liquid, A.H. Nissan, D. Hansen and J.L. Walker, 1963, Chem. Eng. Progr. Symp. Sec. 59, 114.

APPENDIX 1

Thermal Conductivity Equations ($W\ m^{-1}\,°K^{-1}$)

Liquid water[1].

$$k_w = 0.00152T^1 \pm 0.570887$$

Dry Air[1].

$$k_a = 0.00081T^1 + 0.24113$$

Water vapour (conduction)[2].

$$k_{wv} = 0.000082T^1 + 0.015881$$

Ice[1].

$$k_i = 0.0312T^1 + 2.1$$

where T^1 = temperature in °C.

Water vapour (diffusion)

$$k_d = \lambda(DM/RT)\ (P_T/P_T - P)\ (dP/dT)$$

where λ = latent heat of evaporation

D = diffusion of coefficient

M = molecular weight

T = absolute temperature

P_T = total pressure

P = saturated vapour pressure.

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1. Values from Chemical Rubber Company Handbook of Tables for Applied Engineering Science 2nd Edition, 1973.
2. Values from Chemical Rubber Company Handbook of Chemistry and Physics, 55th Edition, 1974-1975.

APPENDIX 2

Table 1: Temperature Gradient

Mean Temperature = +16.5°C

% Fibre (x)	Thermal Conductivity (K_T) W m^{-1}°K^{-1} (y)			
	$V_T = V_{fs}+V_{as}$	$V_T = V_{fs}+V_{ws}$	$V_T = V_{fp}+V_{ap}$	$V_T = V_{fp}+V_{wp}$
0	0.255	.5968	.0255	.5968
10	0.278	.4624	.0382	.5524
20	.0305	.3773	.0509	.5080
30	.0339	.3187	.0636	.4636
40	.0382	.2759	.0764	.4192
50	.0436	.2432	.0891	.3748
60	.0509	.2174	.1018	.3304
70	.0611	.1966	.1145	.2859
80	.0764	.1794	.1273	.2415
90	.1018	.1650	.1400	.1971
100	.1527	.1527	.1527	.1527

Mean Temperature = -16.5°C

	$V_T = V_{fs}+V_{as}$	$V_T = V_{fs}+V_{is}$	$V_T = V_{fp}+V_{ap}$	$V_T = V_{fp}+V_{ip}$
0	.0228	1.5852	.0228	1.5852
10	.0249	.7695	.0342	1.4403
20	.0273	.5081	.0456	1.2955
30	.0304	.3792	.0569	1.1506
40	.0342	.3025	.0683	1.0058
50	.0391	.2516	.0797	.8609
60	.0456	.2154	.0911	.7161
70	.0547	.1883	.1025	.5712
80	.0683	.1672	.1139	.4264
90	.0911	.1504	.1253	.2815
100	.1367	.1367	.1367	.1367

where $V_T = 1$

APPENDIX 2

Table 2: Temperature and Vapour Gradient

Mean Temperature = +16.5°C

% Fibre (x)	Thermal Conductivity (K_T) W m^{-1}°K^{-1} (y) $V_T = V_{fs} + V_{as} + V_{ds}$	$V_T = V_{fp} + V_{ap} + V_{dp}$
0	.0239	.0251
10	.0261	.0378
20	.0287	.0506
30	.0320	.0634
40	.0360	.0761
50	.0413	.0889
60	.0484	.1017
70	.0583	.1144
80	.0735	.1272
90	.0992	.1399
100	.1527	.1527

Mean Temperature = -16.5°C

	$V_T = V_{fs} + V_{as} + V_{ds}$	$V_T = V_{fp} + V_{ap} + V_{dp}$
0	.0228	.0228
10	.0249	.0342
20	.0274	.0455
30	.0305	.0569
40	.0343	.0683
50	.0391	.0797
60	.0457	.0911
70	.0548	.1025
80	.0684	.1139
90	.0912	.1253
100	.1367	.1367

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DOCUMENT CONTROL DATA - R & D

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1. ORIGINATING ACTIVITY Defence Research Establishment Ottawa National Defence Headquarters Ottawa, Canada, KIA 0Z4	2a. DOCUMENT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. DOCUMENT TITLE
HEAT AND MOISTURE TRANSFER IN CLOTHING SYSTEMS. PART 2. THEORETICAL CONSIDERATION OF THE EFFECTS OF SOME VARIABLES ON THERMAL CONDUCTIVITY

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
TECHNICAL NOTE

5. AUTHOR(S) (Last name, first name, middle initial)

CROW, Rita M.

6 DOCUMENT DATE SEPTEMBER 1975	7a. TOTAL NO. OF PAGES 20	7b NO OF REFS 3

8a. PROJECT OR GRANT NO. 79-01-04	9a ORIGINATOR'S DOCUMENT NUMBER(S) DREO Technical Note No. 75-19
8b. CONTRACT NO.	9b. OTHER DOCUMENT NO.(S) (Any other numbers that may be assigned this document)

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING ACTIVITY

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